Many-body phenomena in ultra-cold Fermi gases in optical resonators
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Dynamical light-matter coupling
In a cavity the back-action of the atoms on the light field becomes relevant. The system can dynamically stabilize exotic states, as supersolids, quasi-crystals, superconductivity and dynamical magnetic fields.

Cavity-induced dynamic Hofstadter butterfly

Objective: realization of artificial gauge fields in superradiant states of hybrid systems of light and neutral matter.

Physics in a nutshell:
1) A space dependent phase, similar to the one acquired by electrons in a magnetic field, is imprinted to the atomic wave-function by the exchange of photons between an external laser and a cavity mode.
2) The magnetic field piercing the unit cell is statically determined by the pumping geometry.

System
- 2D spinless Fermi gas in a square lattice.
- Single cavity mode \( |g_1 \rangle \) is pumped by two transverse running waves \( (\Omega_1 \pm i\Omega_2) \) and restores the hopping along the cavity axis.

Harper-Hofstadter Hamiltonian

\[
H = -t \sum_{\text{legs}} \left[ \frac{\Delta \phi}{\gamma} \right] \left( n + \frac{1}{2} \right) + \mu \sum_{\text{legs}} \left( n + \frac{1}{2} \right) + V_{\text{cavity}}
\]

\[
\Delta \phi = \gamma \sum_{\text{legs}} \left( \frac{\Delta \phi}{\gamma} \right) \left( n + \frac{1}{2} \right)
\]

\[
\mu \equiv \mu \left( \frac{\Delta \phi}{\gamma} \right)
\]

Properties
- Superradiance: the cavity mode is coherently populated by increasing the external pump strength.
- Umklapp processes: for weak magnetic field the phase transition becomes first order due to atom-photon scattering processes that bring particles outside the first Brillouin zone.
- Fractal band structure: the energy spectrum is distorted but preserves the topological nature of the static Hofstadter model.

Method development
Numerical development of a Monte Carlo method for boson-fermion model with dissipation.

Objective: realization of density dependent dynamical artificial gauge fields.

Physics in a nutshell:
1) The phase acquired by electrons in a magnetic field is emulated in hybrid systems of coupled light and neutral matter.
2) The phase of the atomic wave-function is coupled to the respective cavity modes.
3) The magnetic flux piercing one plaquette is dynamically determined by the phase of the cavity fields and the atomic density configuration.

System
- Cascade a degenerate quantum gas in two different internal states is confined in a 1D optical lattice.
- Two cavity modes \( (\Omega_1 \pm i\Omega_2) \) are pumped by two transverse constant pumping lasers \( (\Omega_1 \pm i\Omega_2) \) and respectively restore the hopping along the two legs \( (|g_1 \rangle \) and \( |g_2 \rangle \).

Dynamical ladder

\[
H = -t \sum_{\text{legs}} \left[ \frac{\Delta \phi}{\gamma} \right] \left( n + \frac{1}{2} \right) + \mu \sum_{\text{legs}} \left( n + \frac{1}{2} \right) + V_{\text{cavity}}
\]

\[
\Delta \phi = \gamma \sum_{\text{legs}} \left( \frac{\Delta \phi}{\gamma} \right) \left( n + \frac{1}{2} \right)
\]

\[
\mu \equiv \mu \left( \frac{\Delta \phi}{\gamma} \right)
\]

\[
\phi_c \text{ creates a particle at position in the leg } |g_1 \rangle
\]

\[
\phi_0 \text{ creates a particle at position in the leg } |g_2 \rangle
\]

Properties
- Superradiance
- Dynamical magnetic flux: the redistribution of the particles in the two legs determines the value of the magnetic flux at each filling.
- Phase transition: the system shows a transition from an equally populated photon state (macroscopic chiral current \( \Lambda \)) to a photon imbalanced state (macroscopic charge current \( \Psi \)).
- The photon number measures the current in the system.

Crytatization and pairing of a Fermi gas in a ring cavity

Objective: realization and measurement of Cooper pairs in Fermi gases coupled to a ring cavity.

Physics in a nutshell:
1) Photons are delocalized within the cavity and mediate infinite long-range interactions between atoms.
2) For attractive interactions two fermions near the Fermi surface can bound in a Cooper pair.

Project steps
- Study of the two-body momentum correlation in 1D and entanglement properties of few fermions coupled to two degenerate mode of a ring cavity.
- Study beyond mean field of the competing order between spin waves and superconductivity in a 2D Fermi gas coupled to the photon field of a ring cavity.