Exploring Time in Non-relativistic Quantum Mechanics

David Trillo Fernández*

1.Introduction

Suppose you have a d-dimensional quantum system evolving under a time-independent Hamiltonian. You don't know anything else about it; not even how it interacts with other systems. In particular, this means that you cannot exert control over it (for example, you cannot implement a Z gate).

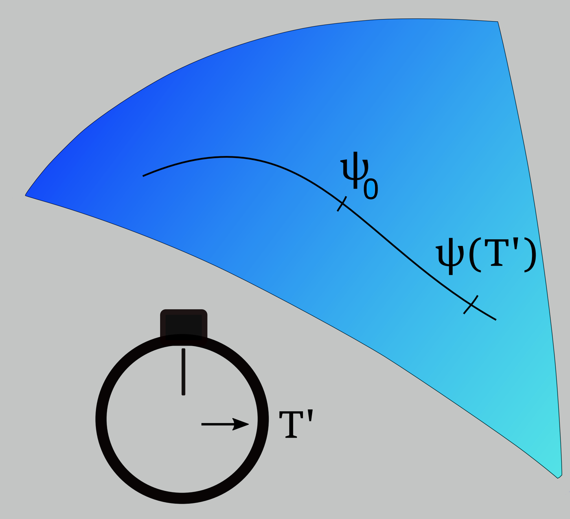

Figure 1: The normal time evolution of a quantum system. After time T' has passed, the system has gone from its initial state $|\psi_0\rangle$ to the state $|\psi(T')\rangle = e^{-iHt}|\psi_0\rangle$.

We want to act on such a system with a protocol of duration T' which projects the system to a state of its past or future evolution curve.

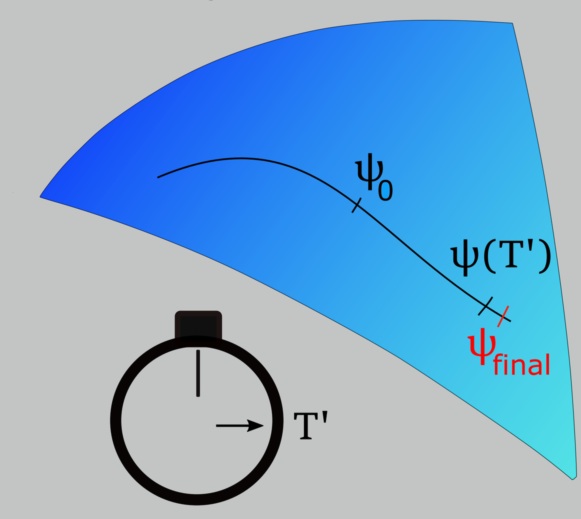

Figure 2: A fast-forwarding protocol

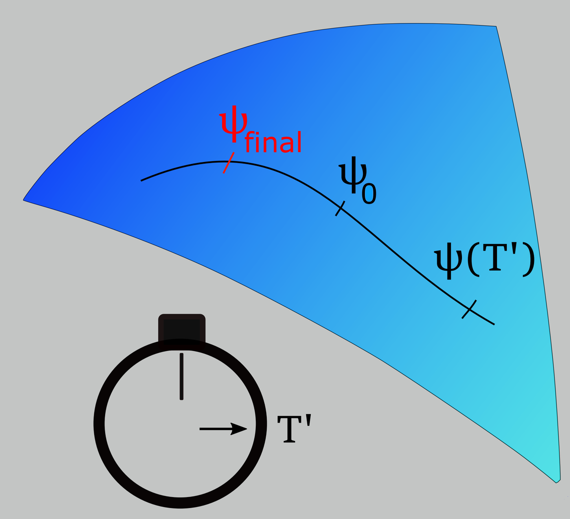

Figure 3: A rewinding protocol

In particular, these protocols cannot be deterministic, but should be universal, non-relativistic and heralded. This is called a time translation and was introduced in [1].

2. The Setup: how to influence an unknown system

In [2] we consider the general setting in which we have n (target) systems which evolve in the same way. We influence them by preparing and sending other systems (probes) to interact with them at specific times. At the end we measure the probes to herald a success. A protocol consists on a particular preparation and measure scheme.

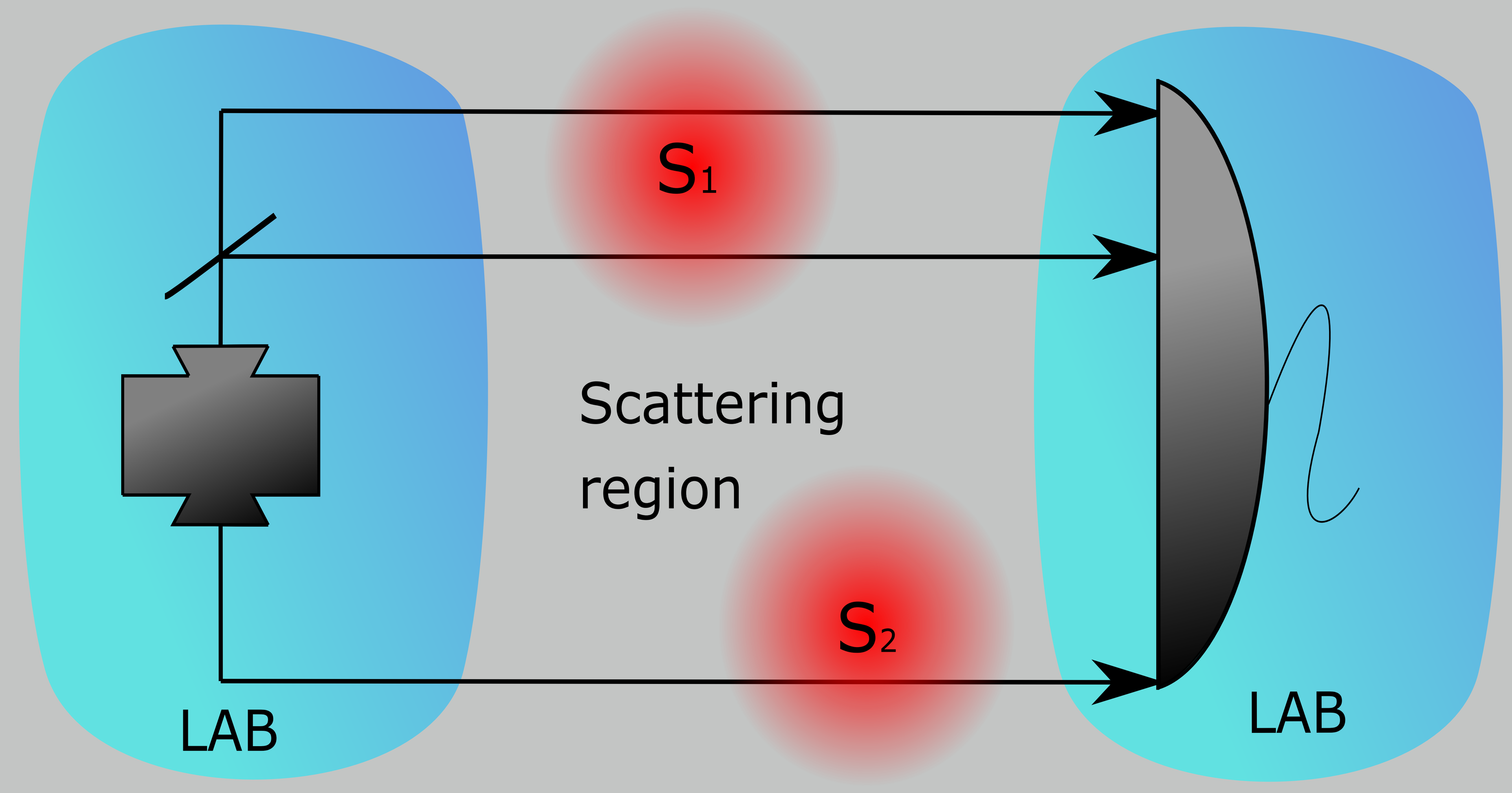

Figure 4: A schematic drawing of the setup. Probes are prepared and measured in a controlled environment. The target systems are sufficiently far apart from each other so that every probe interacts only with one of them.

3. The mathematical formalism: Matrix polynomials

Suppose that we have one system, n probes are prepared in the state $|\psi\rangle_P$ and that each one interacts with the system via a unitary W. Let δ be the time that it takes for a probe and the system to interact. If we send a probe at each of the times $\{t, 2t+\delta, 3t+2\delta, \ldots\}$, the system is evolving freely during t seconds between one probe and the next. Thus, if we post-select on the probes being in the state $|0\ldots0\rangle_P$, the final state of the system will be

$$\langle 0\ldots0|_P\, W^{(k)}(V\otimes 1)\cdots W^{(1)}(V\otimes 1)\,|\phi\rangle_S\,|\psi\rangle_P\,,$$

where $V = e^{-iHt}$ is the time evolution of the system for a fixed t and $W^{(k)}$ is the unitary W acting on the system and the k-th probe. By expanding the state of the probes in an orthonormal basis $\{|k\rangle\}_{k=1}^{d_P}$:

$$|\psi\rangle_P = \sum_{\vec{k}} c_{\vec{k}}\,|k_1\cdots k_n\rangle$$

and calling

$$U_k = \langle 0|_P\, W(V\otimes 1)\,|k\rangle_P\,,$$

we get

$$|\phi_{final}\rangle_S = \left(\sum_{\vec{k}} c_{k_1\cdots k_n} U_{k_1}\cdots U_{k_n}\right)|\phi\rangle_S\,.$$

That is, the state after the protocol is the initial state being acted upon by a homogeneous polynomial on matrix variables. The converse is also true. In general, the correspondence (protocol $\longleftrightarrow$ polynomial) is of the following form:

Number of targets $\longleftrightarrow$ Number of Kronecker products
Dimension of target $\longleftrightarrow$ Size of matrix variables
Dimension of probes $\longleftrightarrow$ Number of variables
Duration of the protocol $\longleftrightarrow$ Degree of the polynomial

There are protocols doing what we intend because of the existence of *central polynomials*. That is, polynomials in matrix variables which are always proportional to the identity [3], and variations of those, which we characterize in [2].

4. Results so far

Let P be a protocol on n copies of a system of dimension d. If at the end of a heralded success, system i is in state $\psi(T_i)$ and the protocol took time T', then we show in [2] that

$$\sum_{i:T_i<0}(d-1)|T_i| + \sum_{j:T_j>0} T_j \leq nT'. \qquad (1)$$

Moreover, this inequality is optimal, meaning that every time translation that satisfies (1) can be asymptotically achieved.

5. Goals and open problems

- Since the protocols are probabilistic, a major question is studying how these probabilities behave. For qubits, we have found in [4] that there are resetting protocols where the probability of success can be as close to 1 as we want. However, it is still unknown for systems of other dimensionality, and for other type of protocols (fast-forwarding, rewinding, etc.).
- This research has already inspired new mathematics to appear, in the form of tensor matrix polynomials [6,7]. Many interesting open problems remain there.
- We would like to apply these techniques to other problems. In particular, getting probabilistic or approximate identity channels in systems as varied as fiber optics or rotations of linear molecules.
- An optical implementation of the protocols described in [1] has already appeared in [5]. However, the protocols in [2] are optimized, and we would like to realize these in collaboration with experimental groups in the University of Vienna.

6. References

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*david.trillo@oeaw.ac.at